Progressive Education Society's
Modern college of Arts, Science & Commerce,
Ganeshkhind, Pune 16 (Autonomous)
End Semester Examination March/April 2025
Faculty: Science and Technology

Program: BScGen03	**Semester VI**	**SET A**
Program(Specific): B.Sc		**Course Type:Core**
Class: T.Y.B.Sc.(Mathematics)		**Max. Marks:35**
Name of the Course: Complex Analysis		**Course Code: 24-MT 361**
Paper No: I		**Time: 2 Hours**

Instructions To the Candidates:
1. There are 3 sections in the question paper. Write each section on separate page.
2. All Sections are compulsory.
3. Figures to the right indicate full marks.
4. Draw a well labelled diagram wherever necessary.

SECTION: A

Q.1) Attempt any **five** of the following. [10 marks]

a) Show that f(z)=$exp(\overline{z})$ is nowhere analytic.

b) Write the function $f(z) = z^2$ in the form $f(z) = u(x, y) + iv(x, y)$.

c) Show that sin(iz)= i sinh(z).

d) Evaluate $\int_0^1 (t + i)\, dt$

e) Find the residue at z=0 of the function $f(z) = \dfrac{z - \sin z}{z}$

f) State Cauchy's Residue theorem.

g) Find $f'(z)$ when $f(z) = \dfrac{z-1}{2z+1}$.

SECTION: B

Q.2) Attempt any **three** of the following. [15 marks]

a) Show that the function $u(x, y) = 2x(1 - y)$ is harmonic. Also find its harmonic conjugate.

b) Show that

(a) $Log(-ei) = 1 - \frac{\pi}{2}i$.

(b) $Log(1 - i) = \frac{1}{2}ln2 - \frac{\pi}{4}i$.

c) Show that $|sinz|^2 = sin^2x + sinh^2y$. Hence find zeros of sinz.

d) Evaluate $\int_C f(z)dz$, if $f(z) = z - 1$ and C is the arc from $z = 0$ to $z = 2$ consisting of

(a) the semicircle $z = 1 + e^{i\theta}$ $(\pi \leq \theta \leq 2\pi)$;

(b) the segment $z = x$ $(0 \leq x \leq 2)$ of the real axis.

e) Write the principal part of the function $f(z) = \dfrac{sin\ z}{z}$ at its isolated singular point and determine whether that point is a pole, a removable singular point or an essential singularity.

SECTION: C

Q.3) Attempt any **one** of the following. [10 marks]

a) Suppose that $f(z) = u(x, y) + i\ v(x, y)$ and that $f'(z)$ exists at a point $z_0 = x_0 + i\ y_0$. Then show that the first order partial derivatives of u and v must exist at (x_0, y_0) and must satisfy the Cauchy Riemann equations $u_x = v_y$ and $u_y = -v_x$ and $f'(z_0) = u_x(x_0, y_0)\ +\ i\ v_x(x_0, y_0)$

b) i) Find residue of $f(z) = z^2\ \sin\ \dfrac{1}{z}$ at the singular point $z = 0$. Hence find the integral $\int_C f(z)dz$ where C is positively oriented unit circle $|z| = 1$.

ii) Let C be the arc of the circle $|z| = 2$, from z=2 to z=2i that lies in the first quadrant. Without evaluating the integral show that

$$\left| \int_C \frac{dz}{z^2 - 1} \right| \leq \frac{\pi}{3}$$

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